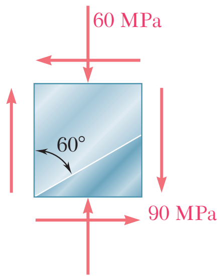
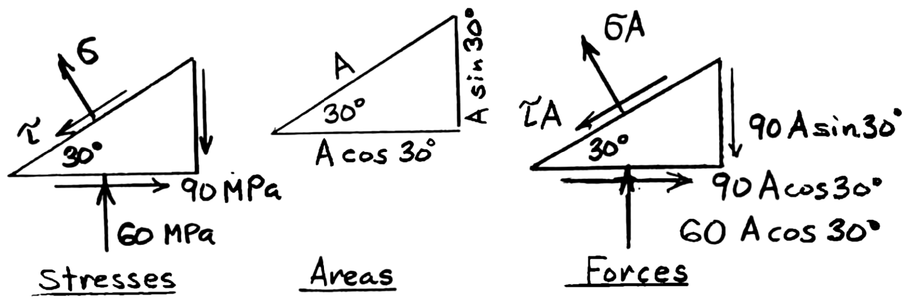


PROBLEM 7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.1A.

SOLUTION



$$+\nwarrow \Sigma F = 0: \quad \sigma A - 90A\sin 30^\circ \cos 30^\circ - 90A\cos 30^\circ \sin 30^\circ + 60A\cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180\sin 30^\circ \cos 30^\circ - 60\cos^2 30^\circ$$

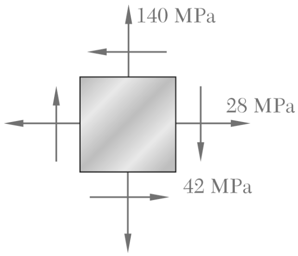
$\sigma = 32.9 \text{ MPa}$ ◀

$$+\swarrow \Sigma F = 0: \quad \tau A + 90A\sin 30^\circ \sin 30^\circ - 90A\cos 30^\circ \cos 30^\circ - 60A\cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90(\cos^2 30^\circ - \sin^2 30^\circ) + 60\cos 30^\circ \sin 30^\circ$$

$\tau = 71.0 \text{ MPa}$ ◀

PROBLEM 7.6



For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$$\sigma_x = 28 \text{ MPa} \qquad \sigma_y = 140 \text{ MPa} \qquad \tau_{xy} = -42 \text{ MPa}$$

(*a*) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-42)}{28 - 140} = 0.750$$

$$2\theta_p = 36.87°$$

$\theta_p = 18.43°, 108.43°$ ◀

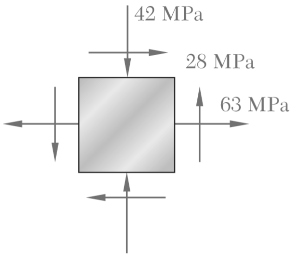
(*b*) $$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{28 + 140}{2} \pm \sqrt{\left(\frac{28 - 140}{2}\right)^2 + (-42)^2}$$

$$= 84 \pm 70$$

$\sigma_{\max} = 154 \text{ MPa}$ ◀

$\sigma_{\min} = 14 \text{ MPa}$ ◀



PROBLEM 7.12

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 63 \text{ MPa} \qquad \sigma_y = -42 \text{ MPa} \qquad \tau_{xy} = 28 \text{ MPa}$$

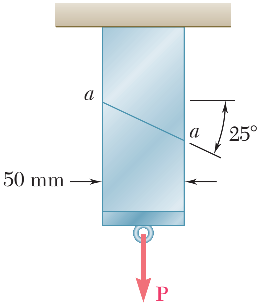
(*a*) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{63+42}{(2)(28)} = -1.875$

$2\theta_s = -61.93°$ $\qquad\qquad$ $\theta_s = -30.96°, 59.04°$ ◀

(*b*) $\tau_{\max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{63+42}{2}\right)^2 + (28)^2} = 59.5 \text{ MPa}$ ◀

(*c*) $\sigma' = \sigma_{\text{ave}} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{63-42}{2} = 10.5 \text{ MPa}$ ◀

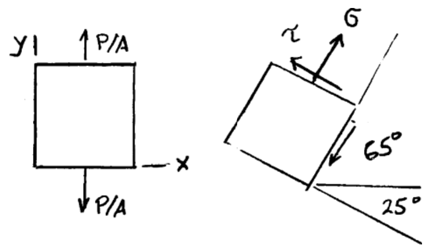


PROBLEM 7.22

Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION

For plane *a-a*, $\theta = 65°$.



$$\sigma_x = 0, \quad \tau_{xy} = 0, \quad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

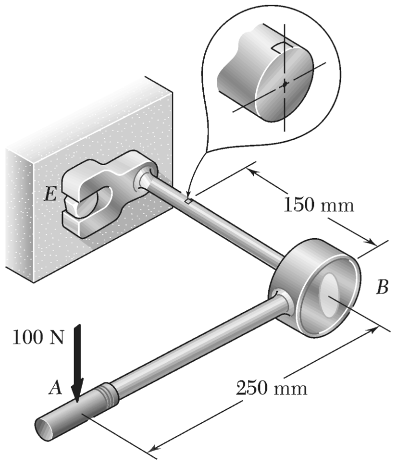
$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^3)}{\sin^2 65°} = 3.90\times10^3\,\text{N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^3)}{\sin 65°\cos 65°} = 6.27\times10^3\,\text{N}$$

Allowable value of P is the smaller one. $P = 3.90$ kN ◀

PROBLEM 7.25



A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 100 N force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the 18 mm diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*.

$$V = 100\text{ N} \quad M = (100)(150) = 15000\text{ N}\cdot\text{mm}$$

$$T = (100)(250) = 25000\text{ N}\cdot\text{mm}$$

Shaft cross section: $d = 18\text{ mm}, \quad c = \frac{1}{2}d = 9\text{ mm}$

$$J = \frac{\pi}{2}c^4 = 10306\text{ mm}^4 \quad I = \frac{1}{2}J = 5153\text{ mm}^4$$

Torsion: $\tau = \frac{Tc}{J} = \frac{(25000)(9)}{10306} = 21.8\text{ N/mm}^2 = 21.8\text{ MPa}$

Bending: $\sigma = \frac{Mc}{I} = \frac{(15000)(9)}{5153} = 26.2\text{ N/mm}^2 = 26.2\text{ MPa}$

Transverse shear: At point *H* stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 26.2\text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 21.8\text{ MPa}$

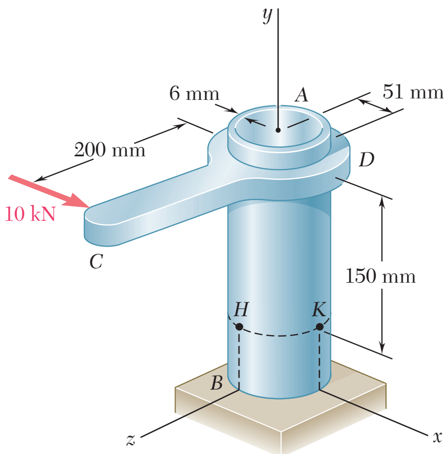
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 13.1\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{13.1^2 + 21.8^2} = 25.4\text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 38.5\text{ MPa} \quad ◀$$

$$\sigma_b = \sigma_{\text{ave}} - R = -12.3\text{ MPa} \quad ◀$$

$$\tau_{\max} = R = 25.4\text{ MPa} \quad ◀$$



PROBLEM 7.26

The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6 \text{ mm}^4$$

$$= 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$$

Force-couple system at center of tube in the plane containing points H and K:

$$F_x = 10 \text{ kN}$$

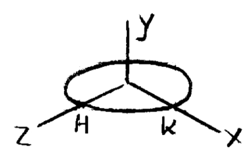
$$= 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3})$$

$$= 2000 \text{ N}\cdot\text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3})$$

$$= -1500 \text{ N}\cdot\text{m}$$



Torsion: At point K, place local x-axis in negative global z-direction.

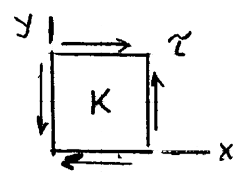
$$T = M_y = 2000 \text{ N}\cdot\text{m}$$

$$c = r_o = 51 \times 10^{-3} \text{ m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6}$$

$$= 24.37 \times 10^6 \text{ Pa}$$

$$= 24.37 \text{ MPa}$$



PROBLEM 7.26 *(Continued)*

Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point *K*.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^6 \text{ Pa} = 36.56 \text{ MPa}$$

Point *K* lies on compression side of neutral axis.

$$\sigma_y = -36.56 \text{ MPa}$$

Total stresses at point *K*:

$$\sigma_x = 0, \quad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$$

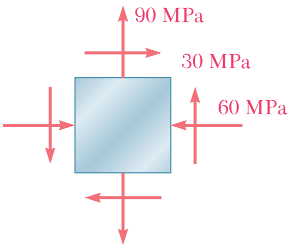
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46 \qquad \sigma_{\max} = 12.18 \text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46 \qquad \sigma_{\min} = -48.7 \text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.5 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.36

Solve Prob. 7.14, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$

$$\sigma_y = 90 \text{ MPa},$$

$$\tau_{xy} = 30 \text{ MPa}$$

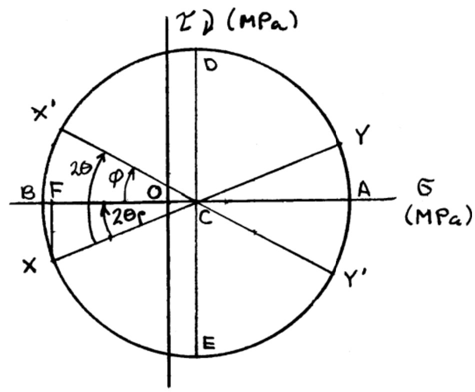
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

<u>Plotted points for Mohr's circle</u>:

$$X: (-60 \text{ MPa}, -30 \text{ MPa})$$

$$Y: (90 \text{ MPa}, 30 \text{ MPa})$$

$$C: (15 \text{ MPa}, 0)$$



$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$

$$2\theta_p = 21.80° \quad \theta_p = 10.90° \circlearrowright$$

$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

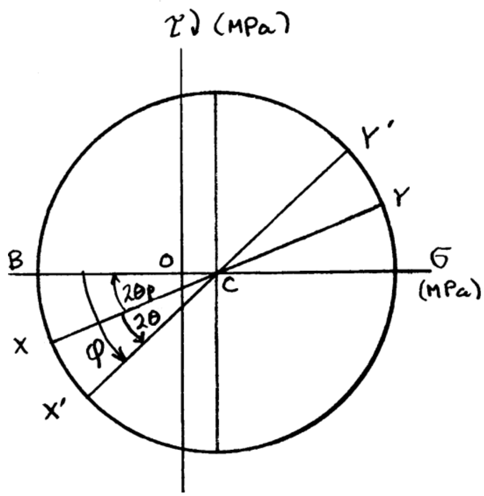
(*a*) $\underline{\theta = 25°}$ $\circlearrowright$. $\quad 2\theta = 50° \circlearrowright$

$$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20° \circlearrowright$$

$\sigma_{x'} = \sigma_{\text{ave}} - R \cos\varphi$ $\qquad\qquad$ $\sigma_{x'} = -56.2 \text{ MPa}$ ◀

$\tau_{x'y'} = -R \sin\varphi$ $\qquad\qquad$ $\tau_{x'y'} = -38.2 \text{ MPa}$ ◀

$\sigma_{y'} = \sigma_{\text{ave}} + R \cos\varphi$ $\qquad\qquad$ $\sigma_{y'} = 86.2 \text{ MPa}$ ◀



(*b*) $\underline{\theta = 10°}$ ↺. $2\theta = 20°$ ↺

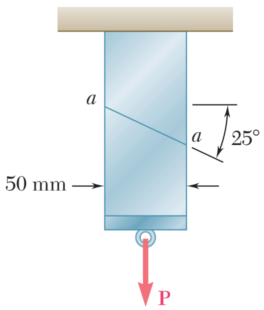
$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80°$ ↺

$\sigma_{x'} = \sigma_{\text{ave}} - R\cos\varphi$ $\sigma_{x'} = -45.2 \text{ MPa}$ ◀

$\tau_{x'y'} = R\sin\varphi$ $\tau_{x'y'} = 53.8 \text{ MPa}$ ◀

$\sigma_{y'} = \sigma_{\text{ave}} + R\cos\varphi$ $\sigma_{y'} = 75.2 \text{ MPa}$ ◀

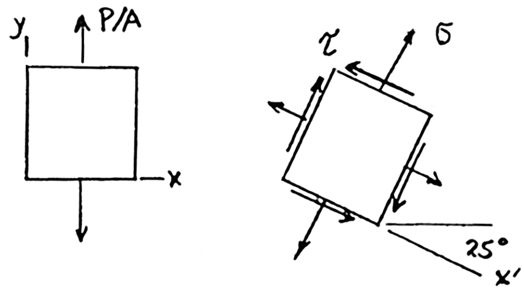
PROBLEM 7.44

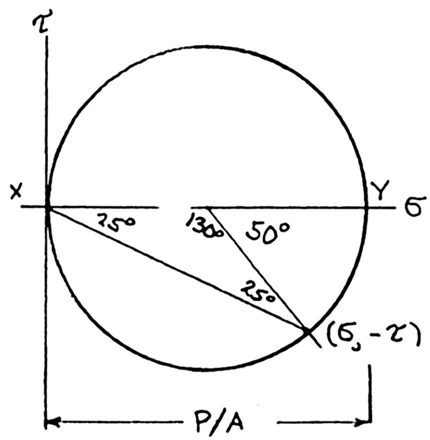


Solve Prob. 7.22, using Mohr's circle.

PROBLEM 7.22 Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION





$$\sigma_x = 0$$

$$\tau_{xy} = 0$$

$$\sigma_y = P/A$$

$$A = (50 \times 10^{-3})(80 \times 10^{-3})$$

$$= 4 \times 10^{-3}\,\text{m}^2$$

$$\sigma = \frac{P}{2A}(1 + \cos 50°)$$

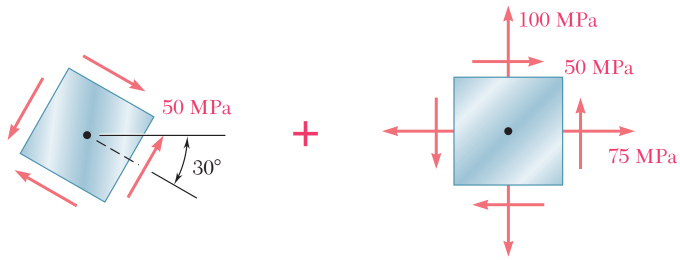
$$P = \frac{2A\sigma}{1 + \cos 50°}$$

$$P \leq \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50°}$$

$$P \leq 3.90 \times 10^3\,\text{N}$$

$$\tau = \frac{P}{2A}\sin 50° \quad P = \frac{2A\tau}{\sin 50°} \leq \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50°} = 6.27 \times 10^3\,\text{N}$$

Choosing the smaller value, $P = 3.90$ kN ◀

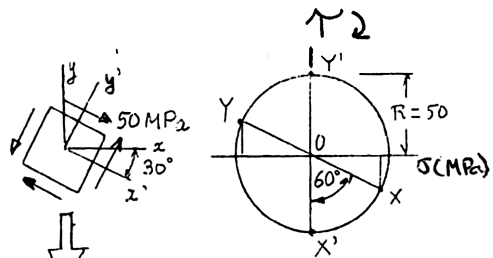


PROBLEM 7.55

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

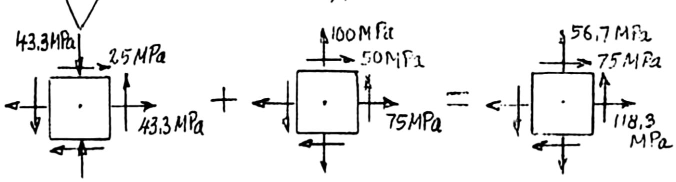
Consider the state of stress on the left. We shall express it in terms of horizontal and vertical components.



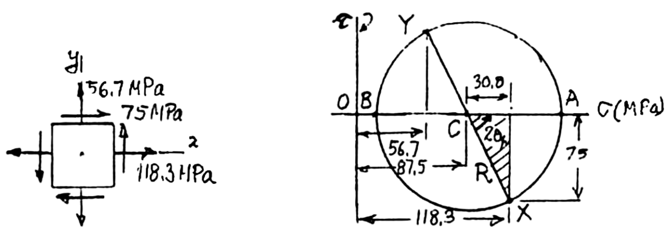
$$\sigma_x = 50\cos 30° = 43.30$$

$$\sigma_y = -43.30$$

$$\tau_{xy} = 50\sin 30° = 25.0$$



Principal axes and principal stress:



$$\sigma_{ave} = \frac{1}{2}(118.3 + 56.7) = +87.5$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{1}{2}(118.3 - 56.7) = 30.8$$

$$R = \sqrt{(30.8)^2 + (75)^2} = 81.08$$

$$\tan 2\theta_p = \frac{75}{30.8} \qquad 2\theta_p = 67.67°$$

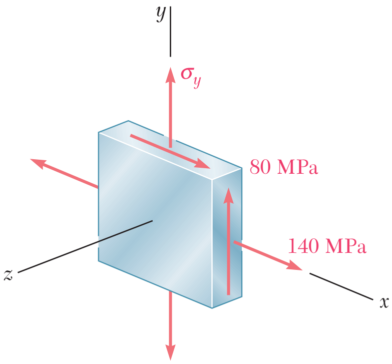
$\theta_p = 33.8°$ ↺, and $123.8°$ ↺ ◀

$$\sigma_{max} = \sigma_{ave} + R = 87.5 + 81.08$$

$\sigma_{max} = +168.6$ MPa ◀

$$\sigma_{min} = \sigma_{ave} - R = 87.5 - 81.08$$

$\sigma_{min} = +6.42$ MPa ◀

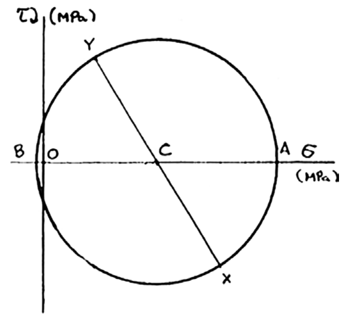


PROBLEM 7.68

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(*a*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 40 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 184.34 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = -4.34 \text{ MPa} \quad \text{(min)}$$

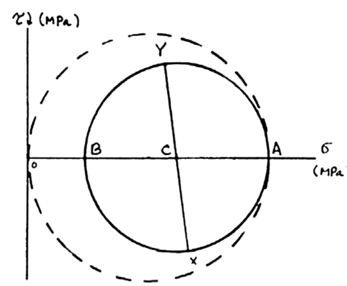
$$\sigma_c = 0$$

$$\tau_{\max\text{(in-plane)}} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.3 \text{ MPa}$$

$\tau_{\max} = 94.3 \text{ MPa}$ ◀

(*b*) $\sigma_x = 140 \text{ MPa}, \quad \sigma_y = 120 \text{ MPa}, \quad \tau_{xy} = 80 \text{ MPa}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 210.62 \text{ MPa} \quad \text{(max)}$$

$$\sigma_b = \sigma_{\text{ave}} - R = 49.38 \text{ MPa}$$

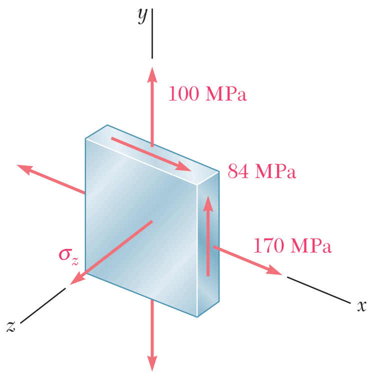
$$\sigma_c = 0 \quad \text{(min)}$$

$$\sigma_{\max} = \sigma_a = 210.62 \text{ MPa} \quad \sigma_{\min} = \sigma_c = 0$$

$$\tau_{\max\text{(in-plane)}} = R = 86.62 \text{ MPa}$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 105.3 \text{ MPa}$$

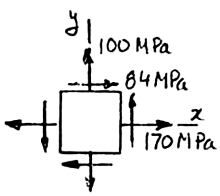
$\tau_{\max} = 105.3 \text{ MPa}$ ◀

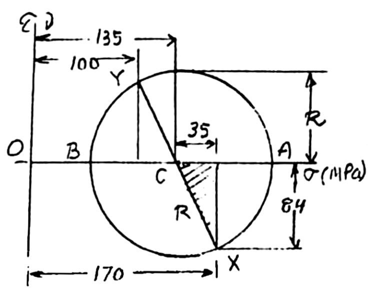


PROBLEM 7.71

For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +60$ MPa, (*c*) $\sigma_z = -60$ MPa.

SOLUTION





$$\sigma_{ave} = \frac{1}{2}(170 + 100) = 135$$

$$\frac{1}{2}(\sigma_x - \sigma_y) = \frac{1}{2}(170 - 100) = 35$$

$$R = \sqrt{(35)^2 + (84)^2} = 91$$

$$\sigma_A = 135 + 91 = 226 \text{ MPa}$$

$$\sigma_B = 135 - 91 = 44 \text{ MPa}$$

(*a*) $\underline{\sigma_z = 0.}$ Point *Z* corresponding to the *z* axis is located at *O*, outside the circle drawn through *A* and *B*. The largest of the 3 Mohr's circles is the circle through *O* and *A*. We have

$$\tau_{max} = \frac{1}{2}(OA) = \frac{1}{2}\sigma_A = \frac{1}{2}(226)$$

$\tau_{max} = 113.0$ MPa ◀

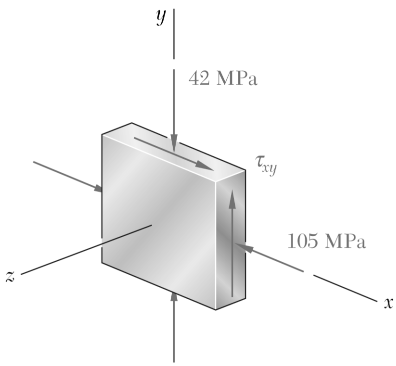
(*b*) $\underline{\sigma_z = +60 \text{ MPa.}}$ Point *Z* is located between *B* and *A*. The largest of the 3 circles is the one drawn through *A* and *B*.

$\tau_{max} = R = 91.0$ MPa ◀

(*c*) $\underline{\sigma_z = -60 \text{ MPa.}}$ Point *Z* is located outside the circle drawn through *A* and *B*. The largest of the 8 Mohr's circles is the circle through *Z* and *A*. We have

$$\tau_{max} = \frac{1}{2}(ZA) = \frac{1}{2}(60 + 226)$$

$\tau_{max} = 143.0$ MPa ◀



PROBLEM 7.75

For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 63 MPa, (*b*) 84 MPa.

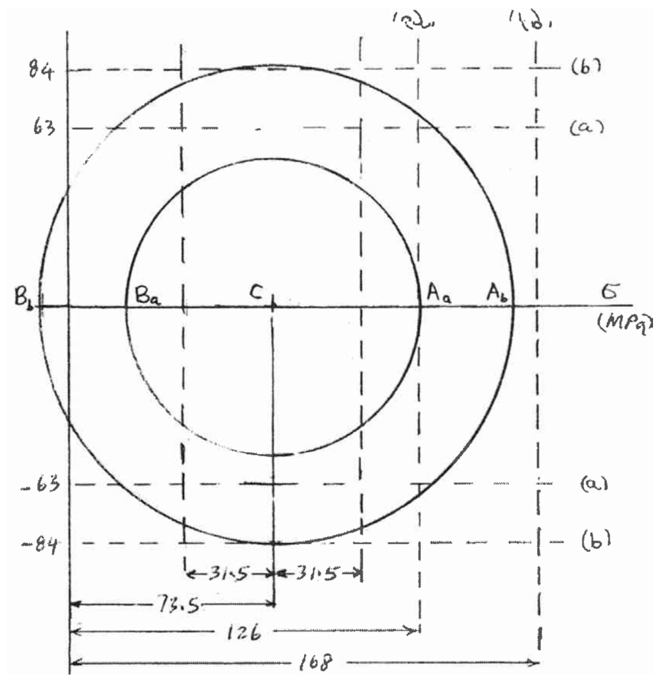
SOLUTION

$$\sigma_x = 105 \text{ MPa} \quad \sigma_y = 42 \text{ MPa}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 73.5 \text{ MPa}$$

$$U = \frac{\sigma_x - \sigma_y}{2} = 31.5 \text{ MPa}$$

τ (MPa)



(*a*) For $\tau_{\max} = 63$ MPa

Center of Mohr's circle lies at point *C*.
Lines marked (*a*) show the limits on $\tau_{\max}$.
Limit on $\sigma_{\max}$ is $\sigma_{\max} = 2\tau_{\max} = 126$ MPa.
For the Mohr's circle $\sigma_a = \sigma_{\max}$ corresponds to point A_a.

$$R = \sigma_a - \sigma_{\text{ave}} = 126 - 73.5 = 52.5 \text{ MPa}$$

$$R = \sqrt{U^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm\sqrt{52.5^2 - 31.5^2} = \pm 42 \text{ MPa}$$ ◀

(*b*) For $\tau_{\max} = 84$ MPa

Center of Mohr's circle lies at point *C*.

$$R = 84 \text{ MPa}$$

$$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm 78.7 \text{ MPa}$$ ◀

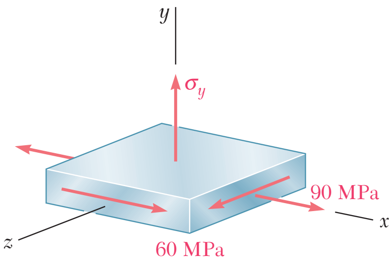
Checking

$$\sigma_a = 73.5 + 84 = 157.5 \text{ MPa}$$

$$\sigma_b = 73.5 - 84 = -10.5 \text{ MPa}$$

$$\sigma_c = 0$$

$$\tau_{\max} = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) = 84 \text{ MPa} \qquad \text{O.K.}$$



PROBLEM 7.79

For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.

SOLUTION

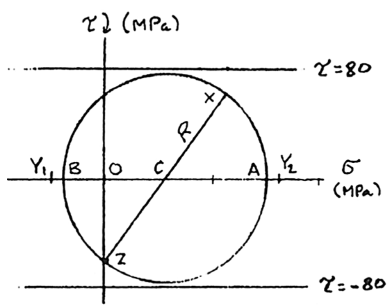
$$\sigma_x = 90\text{ MPa},\quad \sigma_z = 0,\quad \tau_{xz} = 60\text{ MPa}$$

Mohr's circle of stresses in *zx* plane:

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_z) = 45\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75\text{ MPa}$$

$$\sigma_a = \sigma_{\text{ave}} + R = 120\text{ MPa},\quad \sigma_b = \sigma_{\text{ave}} - R = -30\text{ MPa}$$



Assume $\sigma_{\max} = \sigma_a = 120$ MPa.

$$\sigma_y = \sigma_{\min} = \sigma_{\max} - 2\tau_{\max}$$

$$= 120 - (2)(80)$$

$\sigma_y = -40.0$ MPa ◀

Assume $\sigma_{\min} = \sigma_b = -30$ MPa.

$$\sigma_y = \sigma_{\max} = \sigma_{\min} + 2\tau_{\max}$$

$$= -30 + (2)(80)$$

$\sigma_y = 130.0$ MPa ◀